# A Note on Trade Policy with Foreign Ownership

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## 1 Introduction

The ownership of the equities of firms become more complicated. Foreign investment liberalization makes the investors be able to acquire the equities of firms in the other nations. A domestic firm financing by equities may not only be owned by the domestic investors, but also partially owned by the foreign investors. With foreign ownership, any firm is never totally owned by the nation itself.

When involved in trade activities, the presence of foreign ownership affects the government's trade policy and social welfare. In the traditional trade literatures, Bhagwati and Brecher (1980), Brecher and Bhagwati (1981), Brecher and Findlay (1983) have studied the welfare implication of foreign ownership on trade policies when firms behave as price-takers. Applying game approach in the strategic trade policy analyses, Lee (1990), Dick (1993) and Welzel (1995) showed that the presence of foreign ownership weakens the governments' subsidization or taxation incentives in the duopolistic market. Long and Soubeyran (2001) studied the cross ownership structure with a number of heterogeneous firms and extended the trade policy analyses to both domestic and foreign firms.

This note also examines the government's strategic trade policy to both domestic and foreign firms. We use a simple third market model and allow for capital liberalization. The foreign firm moves to the host country and a part of its equities is acquired by the host country's residents. The host country implements the trade policy to both domestic and foreign firms when they export to the third market. We examine how the foreign ownership affects the host government's trade policy determination and firms' production pattern.

The remainder of the paper is organized as follows. Section 2 introduces the model setup and the rent-shifting effect of strategic trade policy. Section 3 examines the host government's optimal trade policy into four cases. Section 4 summarizes the results in the four cases with numerical examples under linear demand function. Finally, the conclusion is summed up in Section 5.

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# 2 Basic Model

We consider a third market model. There are two exporting firms, home firm (labeled as $H$) and foreign firm (labeled as $F$). Both firms produce a homogenous product and their outputs denoted as $x_i (i = H, F)$ are exported to a third country, which does not produce but only consume the product in question. Let $X$ denote the total consumption of the third country and $p = P(X)$ its inverse demand function. The market clearing condition requires $X = x_H + x_F$.

We assume with capital liberalization, the home country attracts the foreign firm to invest and locate in its territory. The home country is a host country and implements the export policy to both firms. Following the framework of Brander and Spencer (1985), international trade is modelled as a two-stage game involving government and firms. In the first stage, home country's government determines a specific export subsidy $s_H$ to its own firm, and $s_F$ to the foreign firm. The subsidy profile is denoted as $\mathbf{s} = (s_H, s_F)$. In the second stage, firms engage in quantity competition in the third market.

For simplicity of exposition, we assume firm $i$ has a constant marginal cost $c_i^0$. The subsidy-ridden marginal cost denoted as $c_i$ is expressed by

$$c_i = c_i^0 - s_i \qquad (i = H, F).$$

The profit function of firm $i$ is expressed as

$$\pi_i \overset{\text{def}}{=} \pi^i(x_i, x_j, c_i) = \{P(x_i + x_j) - c_i\}x_i \qquad (i, j = H, F;\ j \neq i). \tag{1}$$

To assure the existence of a Cournot-duopoly equilibrium, we assume:

**Assumption 1** *Given $(s_H, s_F)$, each firm's profit function $\pi^i(x_i, x_j, c_i)$ is strictly concave in its own output, i.e.,*

$$\frac{\partial^2 \pi^i(x_i, x_j, c_i)}{\partial x_i^2} = 2P'(X) + x_i P''(X) < 0.$$

Assumption 1 implies that the profit-maximizing output of each firm given the rival's, if it ever proves to be positive, is characterized by the first-order condition.

Throughout the rest of the paper, we denote the elasticity of the slope of the inverse demand curve as $E(X) \overset{\text{def}}{=} -XP''(X)/P'(X)$ and the market share of firm $i$ as $\theta_i \overset{\text{def}}{=} x_i/X (i = H, F)$. Under duopoly, $\theta_H + \theta_F = 1$ holds. The strict concavity of the profit function can be written as $2 - \theta_i E(X) > 0 (i = H, F)$.

## 2.1 Reaction Function

To represent an equilibrium, we first define each firm's reaction function. Firm $i$'s reaction function, denoted by $R^i(x_j, c_i)$ is a solution to the following first-order condition for maximizing (1) with respect to its own output.

$$0 = \frac{\partial \pi^i(R^i(x_j, c_i), x_j, c_i)}{\partial x_i} = P\left(R^i(x_j, c_i) + x_j\right) - c_i + R^i(x_j, c_i)P'\left(R^i(x_j, c_i) + x_j\right)$$

where the second-order condition is insured by Assumption 1. The reaction function $R^i(x_j, c_i)$ has following properties.

$$R^i_x(x_j, c_i) \stackrel{\text{def}}{=} \frac{\partial R^i(x_j, c_i)}{\partial x_j} = -\frac{1 - \theta_i E(X)}{2 - \theta_i E(X)} \begin{cases} < 0 & \text{strategic substitution} \\ > 0 & \text{strategic complementary} \end{cases} \tag{2}$$

$$R^i_c(x_j, c_i) \stackrel{\text{def}}{=} \frac{\partial R^i(x_j, c_i)}{\partial c_i} = \frac{1}{P'(X)(2 - \theta_i E(X))} < 0 \tag{3}$$

where use was made of the implicit function theorem in (1) and Assumption 1. (2) implies when the products of both firms are strategic substitutes (or complements), the reaction curves are downward (or upward) sloping. (3) shows that an increase in the marginal cost $c_i$ makes firm $i$'s reaction curve move inward.

To make the comparative statics in the succeeding discussion sensible, we assume that the equilibrium is globally stable in the standard Cournot output adjustment process, which requires

$$\Delta \stackrel{\text{def}}{=} 1 - R^i_x(x_j, c_i) R^j_x(x_i, c_j) = \frac{3 - E(X)}{(2 - \theta_i E(X))(2 - \theta_j E(X))} > 0. \tag{4}$$

Given Assumption 1, it is equivalent to assume as following.

**Assumption 2** *For all the relevant equilibria, there holds* $E(X) < 3$.

Let us denote $x^N_i(c_i, c_j)$ as firm $i$'s equilibrium output. It should satisfy:

$$x^N_i(c_i, c_j) = R^i\left(x^N_j(c_j, c_i), c_i\right) \qquad (i, j = H, F; j \neq i). \tag{5}$$

Accordingly, $X^N(c_H, c_F) \stackrel{\text{def}}{=} x^N_H(c_H, c_F) + x^N_F(c_F, c_H)$ denotes the equilibrium total output and $\hat{\pi}^i(c_i, c_j) \stackrel{\text{def}}{=} \pi^i\left(x^N_i(c_i, c_j), x^N_j(c_j, c_i), c_i\right)$ the equilibrium profit of firm $i$.

## 2.2 Standard Strategic Export Subsidy Policies

We first review the results of the standard strategic export subsidy policies á la Brander and Spencer (1985), i.e., each exporting country has a strategic incentive to subsidize its own exports.

Differentiation of (5) with respect to $c_i$ yields:

$$\begin{pmatrix} 1 & -R^i_x(x^N_j, c_i) \\ -R^j_x(x^N_i, c_j) & 1 \end{pmatrix} \begin{pmatrix} \partial x^N_i / \partial c_i \\ \partial x^N_j / \partial c_i \end{pmatrix} = \begin{pmatrix} R^i_c(x^N_j, c_i) \\ 0 \end{pmatrix}.$$

Solving for the above equation yields the comparative statics on the subsidy-ridden duopoly equilibrium with respect to a change in the effective marginal cost, $c_i$.

$$\frac{\partial x^N_i}{\partial c_i} = \frac{R^i_c(x^N_j(c_j, c_i), c_i)}{\Delta} = \frac{2 - \theta_j E(X)}{P'(X)(3 - E(X))} < 0, \tag{6}$$

$$\frac{\partial x^N_j}{\partial c_i} = R^j_x(x^N_i(c_i, c_j), c_j) \frac{\partial x^N_i}{\partial c_i} = -\frac{1 - \theta_j E(X)}{P'(X)(3 - E(X))} \begin{cases} > 0 & \textit{strategic substitution} \\ < 0 & \textit{strategic complementary} \end{cases}, \tag{7}$$

where use was made of (3)(4) and Assumption 1-2. The total output changes as follows:

$$\frac{\partial X^N(c_H, c_F)}{\partial c_i} = \left\{1 + R_x^j(x_i^N, c_j)\right\} \frac{\partial x_i^N(c_H, c_S)}{\partial c_i} = \frac{1}{P'(X)(3 - E(X))} = \frac{\partial X^N}{\partial c_j} < 0, \tag{8}$$

where use was made of (6) and Assumption 2.

The equilibrium profit of each firm should change as expressed by:

$$\frac{\partial \hat{\pi}^i(.)}{\partial c_i} = x_i^N P'(X^N) \frac{\partial x_j^N}{\partial c_i} - x_i^N < 0, \tag{9}$$

$$\frac{\partial \hat{\pi}^j(.)}{\partial c_i} = x_j^N P'(X^N) \frac{\partial x_i^N}{\partial c_i} > 0, \tag{10}$$

where use was made of (6) and (7) in the case of strategic substitution. The subsidy provision to the domestic firm shifts the duopoly rents from the rival firm to the domestic firm. The rent-shifting effect increases the domestic firm's profit and reduces the rival ones.

# 3 Optimal Trade Policy with Foreign Ownership

We assume that a part of foreign firm's equities is owned by the home country's residents. With foreign ownership, home country's welfare $W^H$ is measured by the sum of the fractional profits of both firms accruing to the home residents, less the total subsidy payments to both firms. It is expressed as below.

$$W^H(s_H, s_F; \sigma) = \hat{\pi}^H(c_H^0 - s_H, c_F^0 - s_F) + \sigma\hat{\pi}^F(c_F^0 - s_F, c_H^0 - s_H) - \sum_{k=H,F} s_k x_k^N(c_H^0 - s_H, c_F^0 - s_F)$$

where $\sigma \in (0, 1)$ denotes the ratio of foreign firm's equities owned by the home residents. The value of $\sigma$ is assumed to be exogenously given.

We further assume:

**Assumption 3** *The welfare function of home country is strictly concave in the export subsidies* $(s_H, s_F)$.

$$\frac{\partial^2 W^H(s_H, s_F; \sigma)}{\partial s_H^2} < 0 \quad , \quad \frac{\partial^2 W^H(s_H, s_F; \sigma)}{\partial s_F^2} < 0.$$

Solving for the optimal subsidy rates, the first-order conditions of welfare function are shown as below.

$$0 = \frac{\partial W^H(s_H, s_F; \sigma)}{\partial s_H} = -\frac{\partial \hat{\pi}^H}{\partial c_H} - \sigma\frac{\partial \hat{\pi}^F}{\partial c_H} - x_H^N + s_H\frac{\partial x_H^N}{\partial c_H} + s_F\frac{\partial x_F^N}{\partial c_H}, \tag{11}$$

$$0 = \frac{\partial W^F(s_H, s_F; \sigma)}{\partial s_F} = -\frac{\partial \hat{\pi}^H}{\partial c_F} - \sigma\frac{\partial \hat{\pi}^F}{\partial c_F} - x_F^N + s_H\frac{\partial x_H^N}{\partial c_F} + s_F\frac{\partial x_F^N}{\partial c_F}. \tag{12}$$

According to the government subsidy provision, we divide into the following four cases to examine the home government's strategic subsidization incentive.

## 3.1 Case 1: Unilateral subsidy to the home firm, s = $(s_H, 0)$

### 3.1.1 Optimal subsidy for welfare maximization

In the first case, home government only subsidizes its domestic firm. Setting $s_F = 0$ in (11), we get

$$0 = \frac{\partial W^H(s_H, 0; \sigma)}{\partial s_H} = -\frac{\partial \hat{\pi}^H}{\partial c_H} - \sigma \frac{\partial \hat{\pi}^F}{\partial c_H} - x_H^N + s_H \frac{\partial x_H^N}{\partial c_H}. \tag{13}$$

The home country's optimal subsidy denoted as $s_H^L$ yields

$$\begin{aligned} s_H^L(\sigma) &= \frac{\dfrac{\partial \hat{\pi}^H}{\partial c_H} + \sigma \dfrac{\partial \hat{\pi}^F}{\partial c_H} + x_H^N}{\partial x_H^N / \partial c_H} \\ &= x_H^N P'(X) R_x^F + \sigma x_F^N P'(X). \end{aligned} \tag{14}$$

The first term in (14) represents the positive (in the case of strategic substitution) profit effect of the domestic firm. The second term represents the negative profit effect of the foreign firms.

Using the implicit function theorem in (13), we get the following result.

$$\frac{\partial s_H^L(\sigma)}{\partial \sigma} = -\frac{\frac{\partial^2 W^H}{\partial s_H \partial \sigma}}{\frac{\partial^2 W^H}{\partial s_H^2}} = \frac{\frac{\partial \hat{\pi}^F}{\partial c_H}}{\frac{\partial^2 W^H}{\partial s_H^2}} < 0$$

Home government has the incentive to subsidize its own firm more (or tax less) when the foreign ownership decreases.

Without foreign ownership, i.e., $\sigma \to 0$, home country actually subsidizes its exports. It is the standard strategic subsidization result in Brander and Spencer (1985).

$$s_H^B = x_H^N P'(X) R_x^F = s_H^L(0) > 0$$

We summarize the above results into the following corollary.

**Corollary 1** *Under strategic substitution, home unilateral subsidy yields*

- $\frac{\partial s_H^L(\sigma)}{\partial \sigma} < 0.$
- $\sigma \to 0 \quad \Longrightarrow \quad s_H^L > 0.$

### 3.1.2 Welfare analysis in subsidization

The welfare change of home country can be divided as following.

$$\Delta W^H = (\Delta \hat{\pi}^H - \Delta S_H) + \sigma \Delta \hat{\pi}^F$$

where $\Delta S_H$ denotes the change of the subsidy payment to the domestic firm. The first term above is the social profit effect of the subsidy which raises the profit of the domestic firm and subsidy payment. The second term is the dividend effect which lowers the foreign firm's profit. These two effects can be rewritten respectively as below.

$$\begin{aligned}\Delta\hat{\pi}^H - \Delta S_H &= \left[x_H^N P'(X)\Delta x_F^N + x_H^N \Delta s_H\right] - (x_H^N \Delta s_H + s_H \Delta x_H^N) \\ &= [P(X) - c_H](-R_x^F)\Delta x_H^N - s_H \Delta x_H^N \qquad (15) \\ \sigma\Delta\hat{\pi}^F &= \sigma x_F^N P'(X)\Delta x_H^N = -\sigma[P(X) - c_F^0]\Delta x_H^N \qquad (16)\end{aligned}$$

In (15), the first term is the strategic effect of the domestic firm through the change of the foreign firm's output. The second term is the subsidy effect. These two terms sum to the social effect of the subsidy which is ambiguous to decide its signum. In (16), the subsidy to the domestic firm reduces the rival firm's output, so the dividend effect from the foreign firm is always negative. The results are summarized in the following tabular.

| when $s_H = 0$ | Strategic Substitution($R_x^j < 0$) | Strategic Complementary($R_x^j > 0$) |
|---|---|---|
| $\Delta\hat{\pi}^H - \Delta S_H$ | $\oplus$ | $\ominus$ |
| $\sigma\Delta\hat{\pi}^F$ | $\ominus$ | $\ominus$ |

Table 1: Welfare Analysis in Subsidization - Case 1

It is clear to conclude that when the products of the two exporting firms are strategic complements, home government has the incentive to tax its domestic exports. However, when the products are strategic substitutes, the optimal trade policy is dependent on the disparity between the social effect and the dividend effect of the subsidy.

## 3.2 Case 2: Unilateral subsidy to the foreign firm, s $= (0, s_F)$

### 3.2.1 Optimal subsidy for welfare maximization

In the second case, home government only subsidizes its partially owned foreign firm. Setting $s_H = 0$ in (12), we get

$$0 = \frac{\partial W^H(0, s_F, \sigma)}{\partial s_F} = -\frac{\partial\hat{\pi}^H}{\partial c_F} - \sigma\frac{\partial\hat{\pi}^F}{\partial c_F} - x_F^N + s_F\frac{\partial x_F^N}{\partial c_F}. \qquad (17)$$

Solving for the home country's optimal unilateral subsidy to the foreign firm denoted as $s_F^L$ yields

$$\begin{aligned}s_F^L(\sigma) &= \frac{\dfrac{\partial\hat{\pi}^H}{\partial c_F} + \sigma\dfrac{\partial\hat{\pi}^F}{\partial c_F} + x_F^N}{\partial x_F^N/\partial c_F}, \\ &= \sigma x_F^N P'(X)R_x^H + \frac{(1-\sigma)x_F^N}{\partial x_F^N/\partial c_F} + x_H^N P'(X). \qquad (18)\end{aligned}$$

The first term in (18) represents the positive (in the case of substitution) profit effect of the foreign firm. Due to the foreign ownership, a part of the duopoly rents shifts to the domestic shareholders. The second term represents the subsidy payment outflow to the domestic shareholders. The third term shows the negative profit effect of the domestic firm.

Using the implicit function theorem in (17), we get the following result.

$$\frac{\partial s_F^L(\sigma)}{\partial \sigma} = -\frac{\frac{\partial^2 W^H}{\partial s_F \partial \sigma}}{\frac{\partial^2 W^H}{\partial s_F^2}} = \frac{\frac{\partial \hat{\pi}^F}{\partial c_F}}{\frac{\partial^2 W^H}{\partial s_F^2}} > 0$$

Home government has the incentive to subsidize the foreign firm more (or tax less when negative) when the shareholding ratio of the foreign firm increases.

Without foreign ownership, i.e., $\sigma \to 0$, the national welfare is simply the domestic firm's profit less subsidy payment. Home government actually taxes the foreign exports to increases both the domestic firm's profit and tax revenues.

$$s_F^L(0) = \frac{x_F^N}{\partial x_F^N / \partial c_F} + x_H^N P'(X) < 0.$$

We summarize the above results into the following corollary.

**Corollary 2** *Under strategic substitution, foreign unilateral subsidy yields*

- $\frac{\partial s_F^L(\sigma)}{\partial \sigma} > 0.$
- $\sigma \to 0 \quad \Longrightarrow \quad s_F^L < 0.$

### 3.2.2 Welfare analysis in subsidization

The welfare change of home country can be divided as following.

$$\Delta W^H = \Delta \hat{\pi}^H + \left[\sigma \Delta \hat{\pi}^F - \Delta S_F\right]$$

where $\Delta S_F$ is the change of the subsidy payment to the foreign firm. Similarly, the first term above is the social profit effect which lowers the domestic firm's profit. The second term is the net dividend effect of the foreign firm exclusive of the subsidy payment. These two effects can be rewritten respectively as below.

$$\Delta \hat{\pi}^H = x_H^N P'(X) \Delta x_F = -[P(X) - c_H^0] \Delta x_F^N \tag{19}$$

$$\begin{aligned} \sigma \Delta \hat{\pi}^F - \Delta S_F &= \sigma \left[x_F^N P'(X) \Delta x_H^N + x_F^N \Delta s_F\right] - (x_F^N \Delta s_F + s_F \Delta x_F^N) \\ &= \sigma [P(X) - c_F)](-R_x^H) \Delta x_F^N - (1-\sigma) x_F^N \Delta s_F - s_F \Delta x_F^N \end{aligned} \tag{20}$$

In Equation (19), the subsidy to the foreign firm reduces the domestic firm's output, so the profit effect of the domestic firm is always negative. In Equation (20), the first term is the strategic effect of the subsidy to the foreign firm. The second term is the foreign ownership effect indicating the subsidy outflow back to the home country. The third term is the subsidy effect. The above three terms sum to the net dividend effect of the subsidy which is ambiguous to decide its signum. They are summarized in the following tabular.

| when $s_F = 0$ | Strategic Substitution($R^j_x < 0$) | Strategic Complementary($R^j_x > 0$) |
|---|---|---|
| $\Delta\hat{\pi}^H$ | $\ominus$ | $\ominus$ |
| $\sigma\Delta\hat{\pi}^F - \Delta S_F$ | $\begin{cases} \oplus & if \quad \sigma \to 1 \\ \ominus & if \quad \sigma \to 0 \end{cases}$ | $\ominus$ |

Table 2: Welfare Analysis in Subsidization - Case 2

When the products of the two exporting firms are strategic complements, home government has the incentive to tax the foreign products. However, when the products are strategic substitutes, the optimal trade policy is dependent on the disparity between the profit effect and the net dividend effect of the subsidy. We have the following corollary.

**Corollary 3** *In the unilatel subsidization cases, when the products of the two exporting firms are strategic complements, home government's optimal trade policy is to tax the exports. This result is independent of foreign ownership.*

### 3.3 Case 3: Discriminatory subsidy, s $= (s_H, s_F)$

In this case, home government subsidizes both domestic and foreign firms. Rewrite (11) (12) into the following equation.

$$\begin{bmatrix} \dfrac{\partial x^N_H}{\partial c_H} & \dfrac{\partial x^N_F}{\partial c_H} \\ \dfrac{\partial x^N_H}{\partial c_F} & \dfrac{\partial x^N_F}{\partial c_F} \end{bmatrix} \begin{bmatrix} s_H \\ s_F \end{bmatrix} = \begin{bmatrix} \dfrac{\partial \hat{\pi}^H}{\partial c_H} + \sigma \dfrac{\partial \hat{\pi}^F}{\partial c_H} + x^N_H \\ \dfrac{\partial \hat{\pi}^H}{\partial c_F} + \sigma \dfrac{\partial \hat{\pi}^F}{\partial c_F} + x^N_F \end{bmatrix}$$

Given both firms' products are positive, the optimal discriminatory subsidy rates denoted as $s^D_H, s^D_F$ can be solved as below.

$$s^D_H(\sigma) = \sigma x^N_F P'(X) - \frac{x^N_F(1-\sigma)\frac{\partial x^N_F}{\partial c_H}}{D} < 0, \tag{21}$$

$$s^D_F(\sigma) = x^N_H P'(X) + \frac{x^N_F(1-\sigma)\frac{\partial x^N_H}{\partial c_H}}{D} < 0, \tag{22}$$

where $D \overset{\text{def}}{=} \dfrac{\partial x^N_H}{\partial c_H}\dfrac{\partial x^N_F}{\partial c_F} - \dfrac{\partial x^N_F}{\partial c_H}\dfrac{\partial x^N_H}{\partial c_F} = \dfrac{1}{(P'(X))^2(3-E(X))} > 0$ and use was made of (6) (7) (9) (10) and Assumption 2.

When home government implements trade policy to both domestic and foreign firms, it has the incentive to tax both firms for the welfare maximization. Since in the international duopoly, subsidization to the exports leads to overcompetition between the firms. The national welfare worsens if the country also owns a fraction of foreign firm. Therefore to mitigate their competition, the government has the incentive to tax the both exports.

The disparity between the two subsidy rates is expressed as following.[1]

$$
\begin{aligned}
s_H^D(\sigma) - s_F^D(\sigma) &= (\sigma x_F^N - x_H^N)P'(X) - \frac{x_F^N(1-\sigma)\frac{\partial X^N}{\partial c_H}}{D} \\
&= (\sigma x_F^N - x_H^N)P'(X) - (1-\sigma)x_F^N P'(X) \\
&= [(2\sigma - 1)x_F^N - x_H^N]P'(X)
\end{aligned}
$$

If $\sigma \leq 1/2$, we can conclude that $s_H^D > s_F^D$. Home government taxes the domestic firm less than the foreign firm.

**Corollary 4** *In the discriminatory subsidy case, given $x_i^N > 0 (i = H, F)$,*

- *Home government has the incentive to tax both firms, i.e., $s_H^D < 0,\ s_F^D < 0$.*
- *If $\sigma \leq 1/2$, $|s_H^D| < |s_F^D|$.*

### 3.4 Case 4: Uniform subsidy, $\mathbf{s} = (s^U, s^U)$

In this case, home country subsidizes both firms with the same subsidy rate. From (11) (12), the optimal uniform subsidy rate can be solved as below.

$$
\begin{aligned}
0 &= \frac{\partial W^H}{\partial s} = \frac{\partial W^H}{\partial s_H} + \frac{\partial W^H}{\partial s_F} \\
&= -\left(\frac{\partial \hat{\pi}^H}{\partial c_H} + \frac{\partial \hat{\pi}^H}{\partial c_F}\right) - \sigma\left(\frac{\partial \hat{\pi}^F}{\partial c_H} + \frac{\partial \hat{\pi}^F}{\partial c_F}\right) - X^N + s\left(\frac{\partial X^N}{\partial c_H} + \frac{\partial X^N}{\partial c_F}\right) \\
&= -x_H^N P'(X) \sum_{k=H,F} \frac{\partial x_F^N}{\partial c_k} - \sigma x_F^N P'(X) \sum_{k=H,F} \frac{\partial x_H^N}{\partial c_k} - (1-\sigma)x_F^N + s \sum_{k=H,F} \frac{\partial X^N}{\partial c_k}
\end{aligned} \tag{23}
$$

where use was made of (9) (10). Using (6)(7), we note

$$
\sum_{k=H,F} \frac{\partial x_H^N}{\partial c_k} = \frac{1 + (\theta_H - \theta_F)E(X)}{P'(X)(3 - E(X))} \quad , \quad \sum_{k=H,F} \frac{\partial x_F^N}{\partial c_k} = \frac{1 + (\theta_F - \theta_H)E(X)}{P'(X)(3 - E(X))}. \tag{24}
$$

To simplify the analysis, we assume as below.

**Assumption 4** *When both firms lower the marginal cost by the same amount simultaneously, the output of each firm will rise, i.e.,*

$$
\sum_{k=H,F} \frac{\partial x_H^N}{\partial c_k} < 0 \quad , \quad \sum_{k=H,F} \frac{\partial x_F^N}{\partial c_k} < 0
$$

[1]Note using (6)(7)(8), $\dfrac{\partial X/\partial c}{D} = P'(X)$.

Thus, the optimal uniform subsidy denoted as $s^U$ yields

$$s^U(\sigma) = \frac{1}{\sum_{k=H,F} \dfrac{\partial X^N}{\partial c_k}} \left[ x_H^N P'(X) \sum_{k=H,F} \frac{\partial x_F^N}{\partial c_k} + \sigma x_F^N P'(X) \sum_{k=H,F} \frac{\partial x_H^N}{\partial c_k} + (1-\sigma) x_F^N \right] < 0 \tag{25}$$

With foreign ownership, home government also has the incentive to tax both firms to let them behave less aggressively.

**Corollary 5** *In the uniform subsidy case, given $x_i > 0 (i = H, F)$,*

- $s^U(\sigma) < 0 \quad$ *if* $\quad \sum_{k=H,F} \dfrac{\partial x_H^N}{\partial c_k} < 0, \sum_{k=H,F} \dfrac{\partial x_F^N}{\partial c_k} < 0.$

# 4 Numerical Examples under Linear Demand Function

Summarize the four cases discussed as following.

$$\begin{array}{llll}
\textit{Case 1:} & \max\limits_{s_H} & W^H(s_H, s_F) & s.t. \;\; s_F = 0 \\
\textit{Case 2:} & \max\limits_{s_F} & W^H(s_H, s_F) & s.t. \;\; s_H = 0 \\
\textit{Case 3:} & \max\limits_{s_H, s_F} & W^H(s_H, s_F) & \\
\textit{Case 4:} & \max\limits_{s_H, s_F} & W^H(s_H, s_F) & s.t. \;\; s_H = s_F
\end{array}$$

We show the numerical results in the four cases under linear demand function. Assume the inverse demand function as $p = 1 - X = 1 - x_H - x_F$, where $1 > c_i^0 (i = H, F)$. Denote $\beta_i = 1 - 2c_i^0 + c_j^0 > 0 (i, j = H, F; j \neq i)$ for positive quantity under pure duopoly without trade intervention. Since $\beta_H - \beta_F = 3(c_F^0 - c_H^0)$ holds, the ratio $\beta = \beta_H / \beta_F$ serves as the indicator of home firm's relative productivity or efficiency to foreign firm. The value of $\beta$ is important in determining the equilibria. To simplify the analysis, we confine ourselves to the equilibria when the cost disparity between the firms is not large.

**Assumption 5** $1/3 < \beta = \beta_H / \beta_F < 3$.

Since Assumptions 1-4 are satisfied under linear demand function, the optimal subsidy rates and firms' equilibrium outputs in the four cases can be derived shown in Table 3. Using Assumption 5 and foreign ownership ratio $\sigma \in (0, 1)$, the signums of some equilibrium results can be determined.

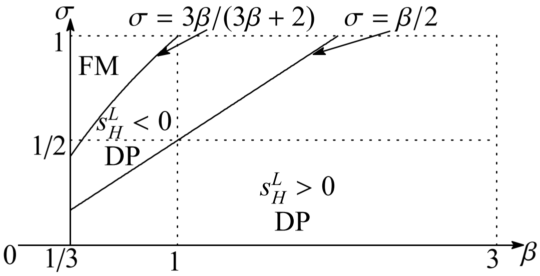
In case 4, the firm outputs are always positive and home government definitely taxes the exports. However, in the other three cases, equilibrium results are dependent on the value of $\beta$ and $\sigma$. Home government may subsidize or tax the exports. The market equilibrium may lead to monopoly since taxation makes the inefficient firm produce at a loss and exit the market. To examine the optimal trade policy and market equilibrium, we depict the results in cases 1-3 in

| | Optimal subsidy | Home Eq. output | Foreign Eq. output |
|---|---|---|---|
| Case 1 | $s_H^L = \dfrac{\beta_H - 2\sigma\beta_F}{2(2-\sigma)}$ | $x_H^N = \dfrac{3\beta_H - \sigma(\beta_H + 2\beta_F)}{3(2-\sigma)}$ | $x_F^N = \dfrac{4\beta_F - \beta_H}{6(2-\sigma)} > 0$ |
| Case 2 | $s_F^L = \dfrac{-(2\beta_H + 3\beta_F) + 4\sigma\beta_F}{2(5-4\sigma)}$ | $x_H^N = \dfrac{3(4\beta_H + \beta_F) - 4\sigma(2\beta_H + \beta_F)}{6(5-4\sigma)} > 0$ | $x_F^N = \dfrac{2(\beta_F - \beta_H)}{3(5-4\sigma)}$ |
| Case 3 | $s_H^D = \dfrac{\beta_H - \beta_F}{6(1-\sigma)}$ <br> $s_F^D = \dfrac{-(\beta_H + 2\beta_F) + 3\sigma\beta_F}{6(1-\sigma)} < 0$ | $x_H^N = \dfrac{3\beta_H - \sigma(2\beta_H + \beta_F)}{6(1-\sigma)}$ | $x_F^N = \dfrac{\beta_F - \beta_H}{6(1-\sigma)}$ |
| Case 4 | $s^U = \dfrac{-(\beta_H + 3\beta_F) + 2\sigma\beta_F}{2(5-\sigma)} < 0$ | $x_H^N = \dfrac{3(3\beta_H - \beta_F) + 2\sigma(\beta_F - \beta_H)}{6(5-\sigma)} > 0$ | $x_F^N = \dfrac{(7\beta_F - \beta_H)}{6(5-\sigma)} > 0$ |

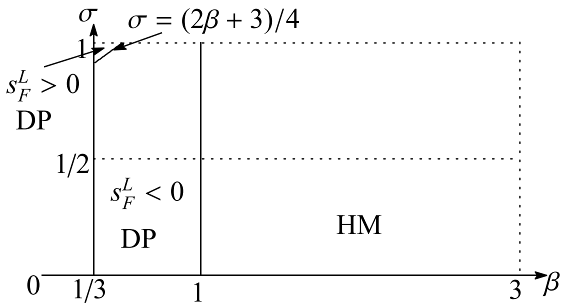
Table 3: Optimal Subsidy Rates and Firms' Equilibrium Outputs

Figure 1. The horizontal axis represents the cost efficiency indicator $\beta \in (1/3, 3)$ and the vertical axis the foreign ownership ratio $\sigma \in (0, 1)$. Each sub-figure shows the area with different trade policy and market equilibrium. We denote DP as duopoly when the two firms manipulate in the market, HM as home monopoly when only home firm produces and FP as foreign monopoly.

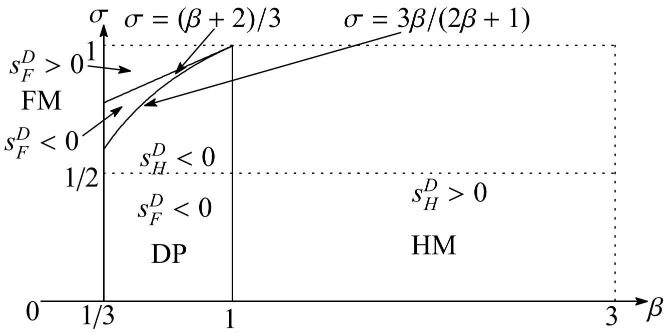
Since $\beta < 1$ means that the foreign firm is more efficient than home firm, the area when $\beta < 1$ may lead to foreign monopoly when the home residents own a large portion of equities of foreign firm. Home government has the incentive to subsidize the foreign firm to increase its production. However, when $\beta > 1$, cases 2-3 lead to home monopoly independent of foreign ownership. Since the productive home firm is totally owned by the home residents, home government subsidizes the home firm to increase the home products. With cost inefficiency and discriminatory trade policy, the foreign firm cannot manipulate in the market. In cases 1-3, home government mostly has the incentive to tax the exports. However, large ratio $\sigma$ may lead to the subsidy to the foreign firm and home efficient production cost $c_H^0$ may lead to the subsidy to the home firm.




(a) Case 1




(b) Case 2




(c) Case 3

Figure 1: Optimal Subsidy and Market Equilibrium

We can also depict the optimal trade policy in the four cases with welfare implication in Figure 2. The horizontal (vertical) axis represents the subsidy rate to the home (foreign) firm. Under linear demand function, $\dfrac{\partial^2 W^H}{\partial s_H \partial s_F} > 0$ holds if $\sigma < \frac{1}{2}$. The reaction curves of $s_H$ and $s_F$ are shown as upward sloping. The iso-welfare curves are depicted and the optimal subsidy rates are shown in the four cases. We find that when the government can discriminate the trade policy to the domestic and foreign firms, the country realizes the highest social welfare.

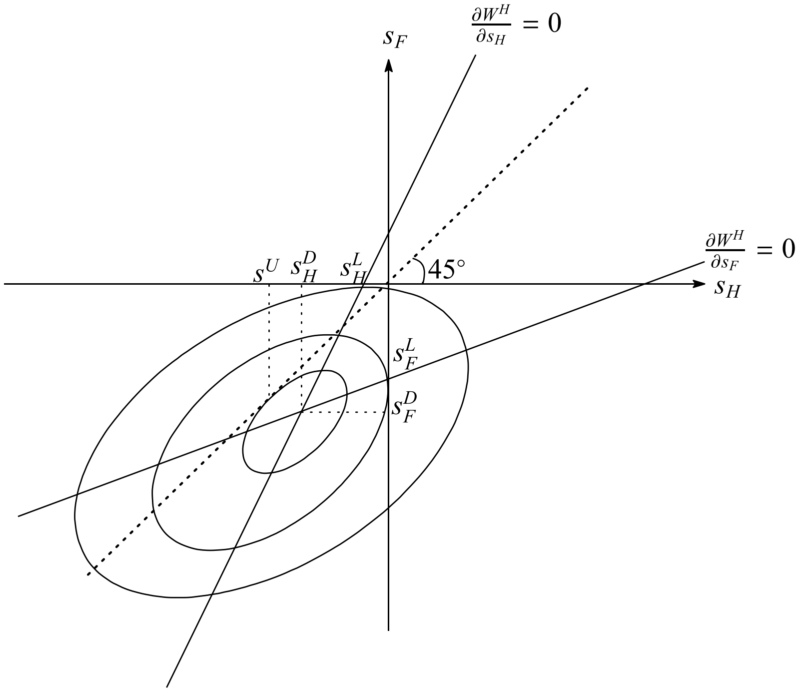



Figure 2: Optimal Subsidy Rates with Iso-Welfare Curves When $\sigma < 1/2$

# 5 Conclusion

This note studies how the foreign ownership affects the host government's trade policy. We examine the four cases to examine the government's strategic policy determination. When the country owns a part of the equities of the foreign firm, the government has the incentive to tax both domestic and foreign firms. Since taxation makes firms compete less aggressively and increase tax revenues, taxing both domestic and foreign firms improves the national welfare. We also show the numerical results under linear demand function and examine how the trade policy and market equilibrium are affected by the foreign ownership and cost efficiency.

This note focuses on the foreign ownership in the four different policy regimes. We do not consider the firm movement with capital liberalization. Actually, when the firms are taxed, they would like to move outside the country to reduce the tax burden. The mobility of firms further affects the government's trade policy and this will leave to the future task.

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